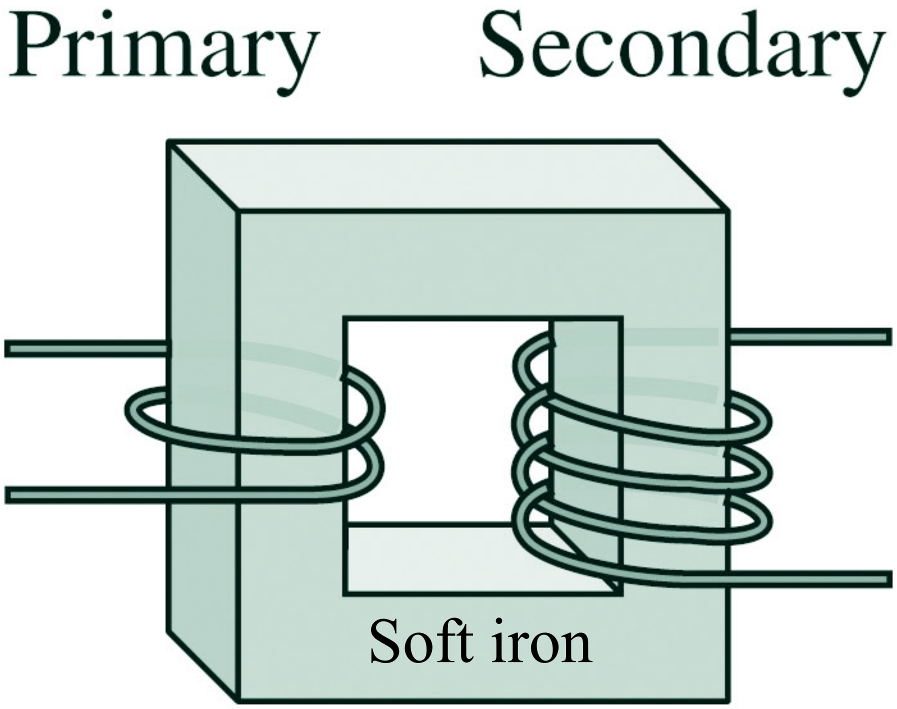
Chapter 29: Electromagnetic Waves
Thursday November 10th

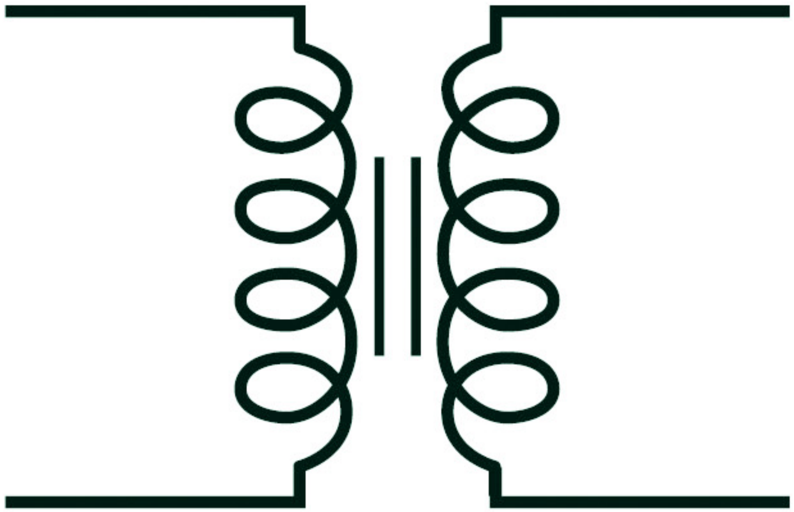
Mini-exam 5 next Thursday (AC circuits and EM waves)
55 unregistered *i*Clickers – any takers?

- Transformers - demo
- Maxwell's equations
- Electromagnetic waves
    - Wave equations
    - Speed of light
    - Relations between quantities
    - Energy flux and intensity

Reading: up to page 515 in the text book (Ch. 28/29)

Transformers





- Flux the same on both sides, but number of turns, *N*, is different
- Total flux through primary and secondary coils depends on N_1 and N_2

$$V_1 = N_1\Phi; \qquad V_2 = N_2\Phi; \qquad \Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

The basic equations of electromagnetism so far.....

Gauss' law:

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{q_{enc}}{\varepsilon_o}$$

Gauss' law for B (no magnetic equivalent of charge):

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

Ampère's law:

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_{\mathrm{o}} I_{enc}$$

Faraday's law:

$$\varepsilon = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}$$

The basic equations of electromagnetism so far.....

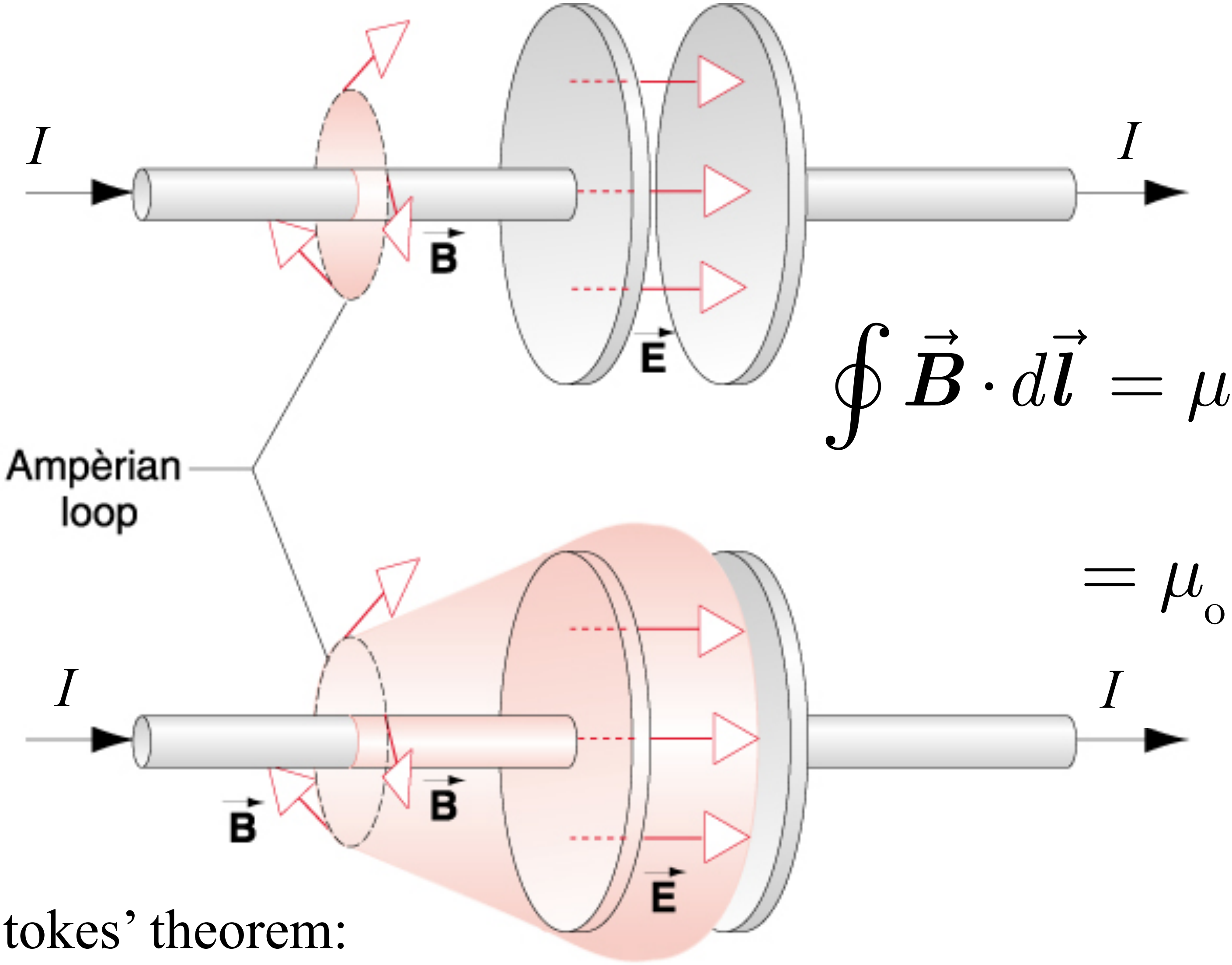
In vacuum:

$$\left.\begin{aligned}\Phi_E &= \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = 0 \\ \Phi_B &= \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0\end{aligned}\right\} \text{Symmetry}$$

$$\left.\begin{aligned}&\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = 0 \\ &\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}\end{aligned}\right\} \text{No Symmetry}$$

Is it possible that a time-varying electric field could produce a magnetic field, thereby restoring symmetry?

Maxwell's displacement current

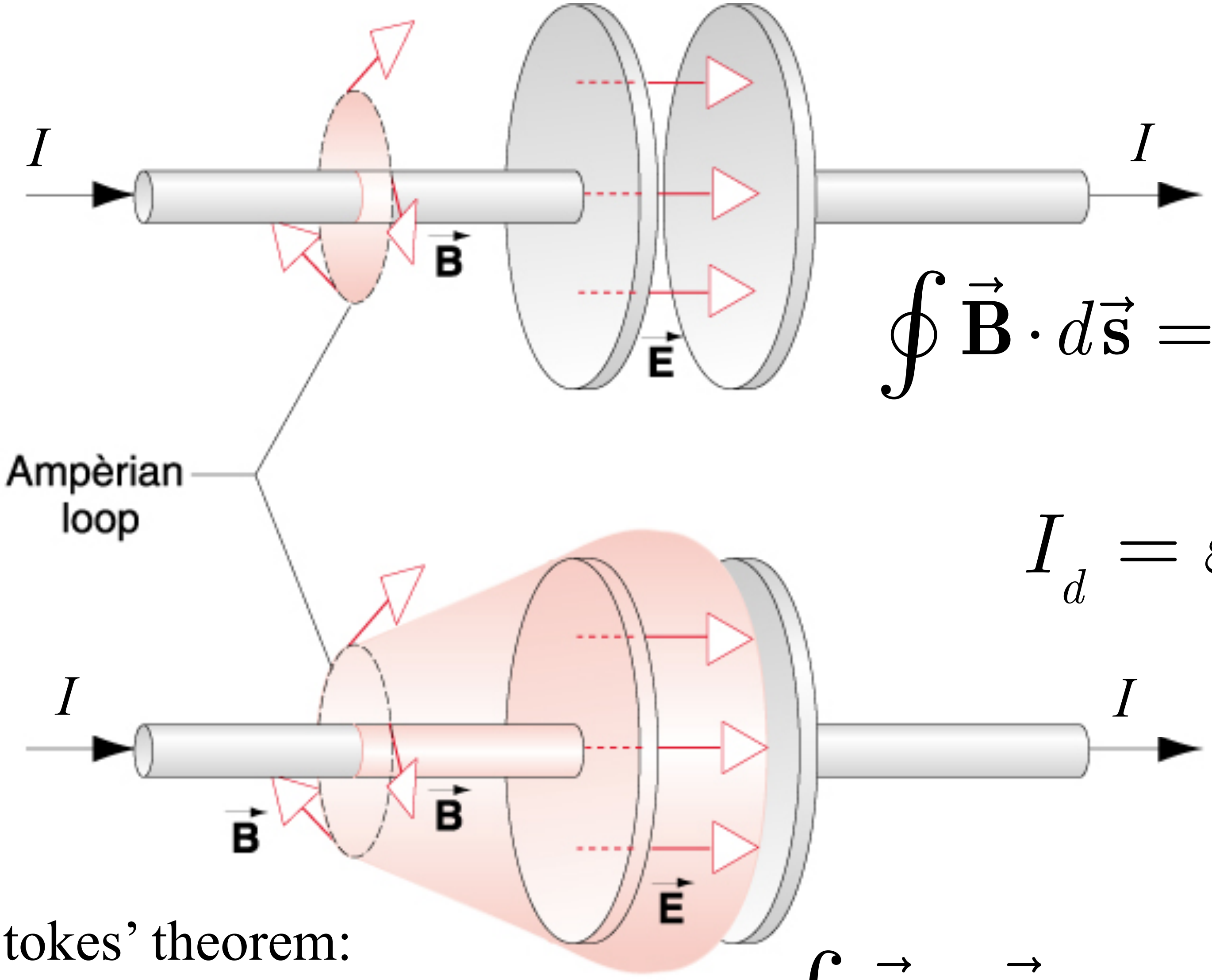


$$\oint \vec{B} \cdot d\vec{l} = \mu_{\text{o}} I_{enc} = \mu_{\text{o}} \Phi_J$$

$$= \mu_{\text{o}} \int \vec{J} \cdot d\vec{A}$$

Stokes' theorem:
The choice of surface
should not matter

Maxwell's displacement current



$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o \left(I + I_d\right)$$

$$I_d = \varepsilon_o \frac{d\Phi_E}{dt}$$

Stokes' theorem:
The choice of surface
should not matter

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

Maxwell's equations

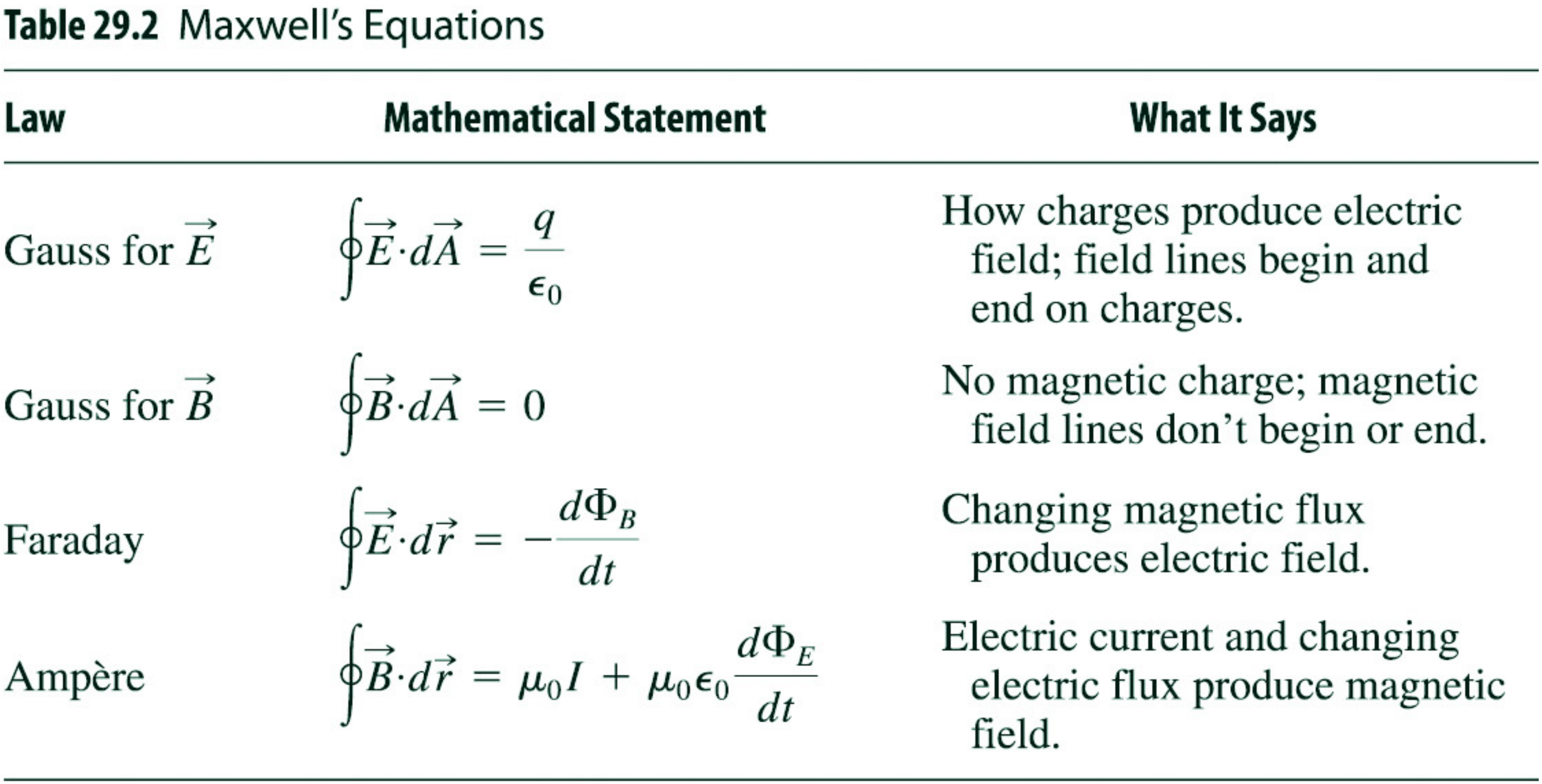


Table 29.2 Maxwell's Equations

Law	Mathematical Statement	What It Says
Gauss for $\vec{E}$	$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$	How charges produce electric field; field lines begin and end on charges.
Gauss for $\vec{B}$	$\oint \vec{B} \cdot d\vec{A} = 0$	No magnetic charge; magnetic field lines don't begin or end.
Faraday	$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$	Changing magnetic flux produces electric field.
Ampère	$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	Electric current and changing electric flux produce magnetic field.

The main thing to note here is the symmetry in the last two equations: a time varying magnetic field produces an electric field; similarly, a time varying electric field produces a magnetic field.

Maxwell's equations in vacuum

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = 0$$

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

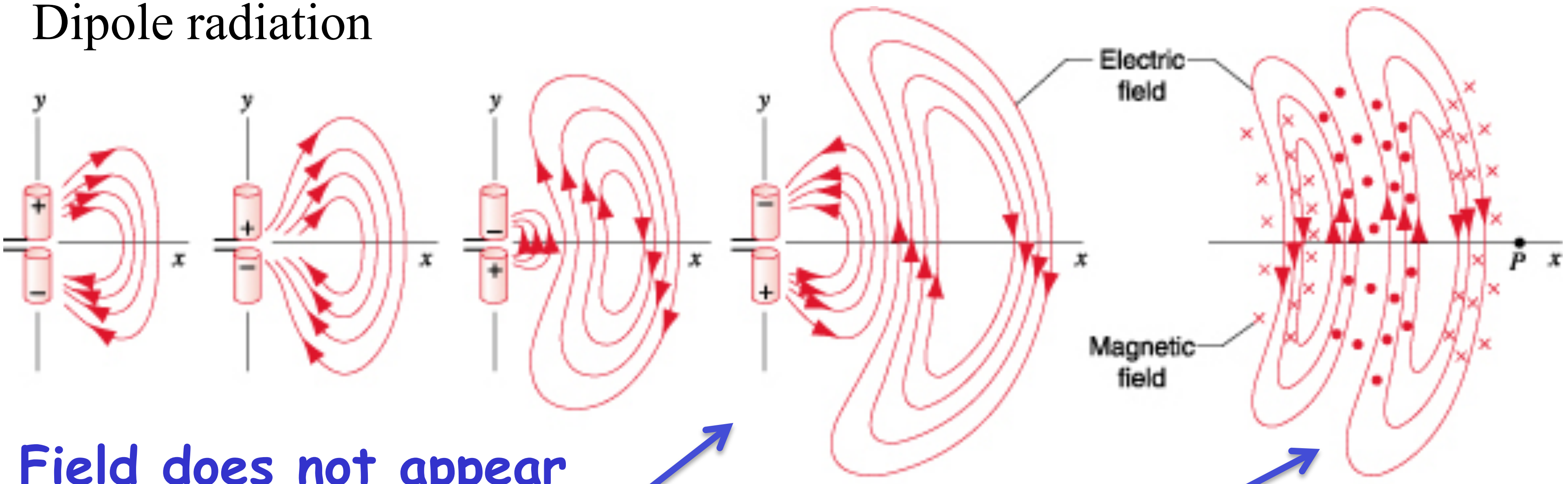
$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}$$

The main thing to note here is the symmetry in the last two equations: a time varying magnetic field produces an electric field; similarly, a time varying electric field produces a magnetic field.

Electromagnetic waves

Dipole radiation

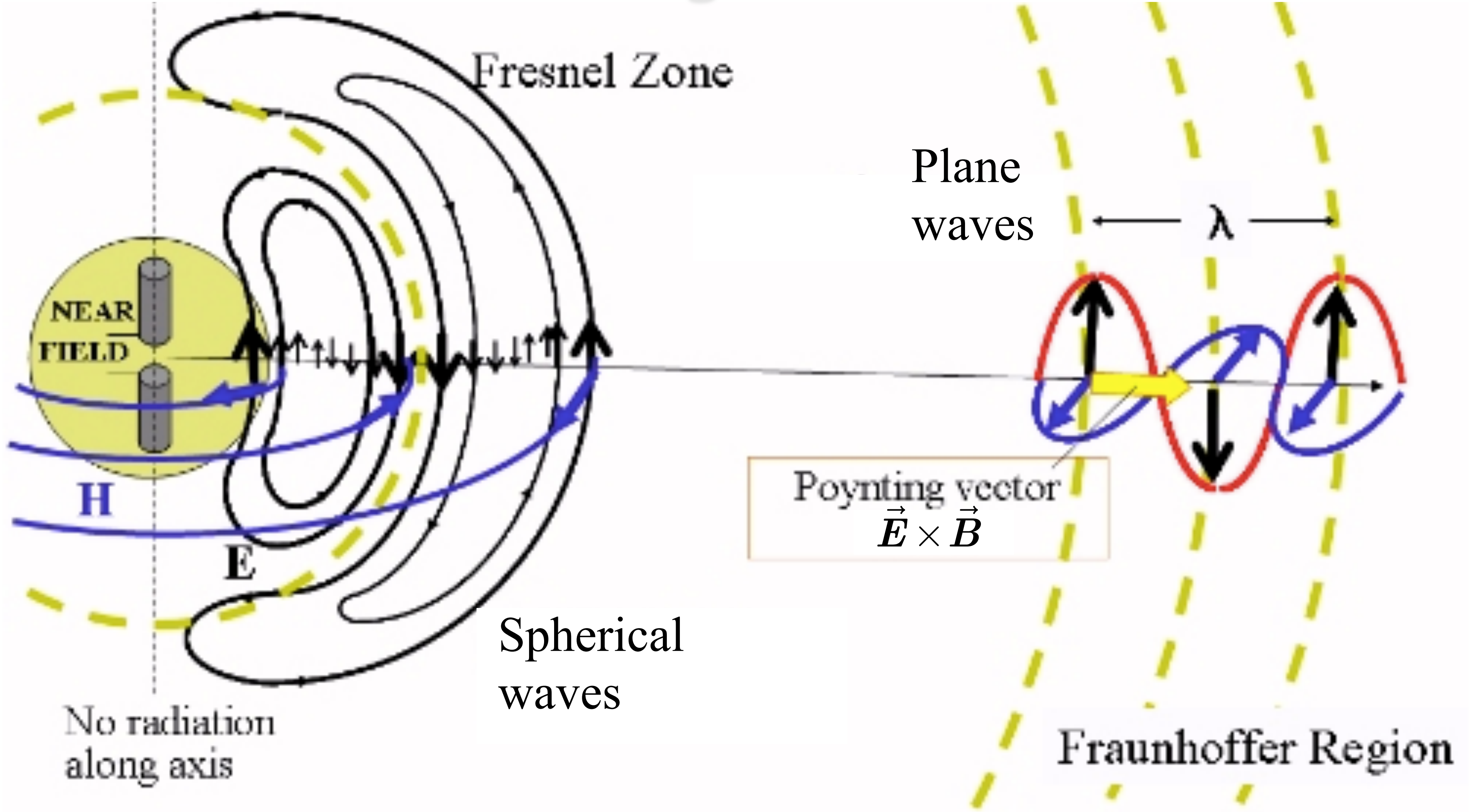


Field does not appear instantaneously

Propagation

Electromagnetic perturbation breaks completely free from the charge/current

Electromagnetic waves



Maxwell's equations guarantee that electric and magnetic fields are perpendicular to each other and perpendicular to the direction of propagation; they are polarized.

Maxwell's equations in vacuum

$$\Phi_E = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = 0$$

$$\Phi_B = \oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_{\mathrm{o}}\varepsilon_{\mathrm{o}} \frac{d\Phi_E}{dt}$$

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi_B}{dt}$$

Stokes' Theorem:
Gives differential form
of Maxwell's equ'ns

$$\left[\nabla \times \vec{\boldsymbol{B}} = \mu_{\mathrm{o}}\varepsilon_{\mathrm{o}} \frac{d\vec{\boldsymbol{E}}}{dt}\right]$$

$$\left[\nabla \times \vec{\boldsymbol{E}} = -\frac{d\vec{\boldsymbol{B}}}{dt}\right]$$

Let there be light!!

Stokes' Theorem:
Gives differential form of Maxwell's equ'ns

$$\nabla \times \vec{\boldsymbol{B}} = \mu_{\mathrm{o}} \varepsilon_{\mathrm{o}} \frac{d\vec{\boldsymbol{E}}}{dt}$$

$$\nabla \times \vec{\boldsymbol{E}} = -\frac{d\vec{\boldsymbol{B}}}{dt}$$

Maxwell's equations in vacuum can be solved simultaneously to give identical differential equations for *E* and *B*:

$$\nabla^2 \vec{\boldsymbol{E}} = \mu_{\mathrm{o}} \varepsilon_{\mathrm{o}} \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2} \qquad \text{and} \qquad \nabla^2 \vec{\boldsymbol{B}} = \mu_{\mathrm{o}} \varepsilon_{\mathrm{o}} \frac{\partial^2 \vec{\boldsymbol{B}}}{\partial t^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The Laplacian differential operator

The Electromagnetic Wave Equations

Let there be light!!

Stokes' Theorem:
Gives differential form of Maxwell's equ'ns

$$\nabla \times \vec{\boldsymbol{B}} = \mu_{\mathrm{o}} \varepsilon_{\mathrm{o}} \frac{d\vec{\boldsymbol{E}}}{dt}$$

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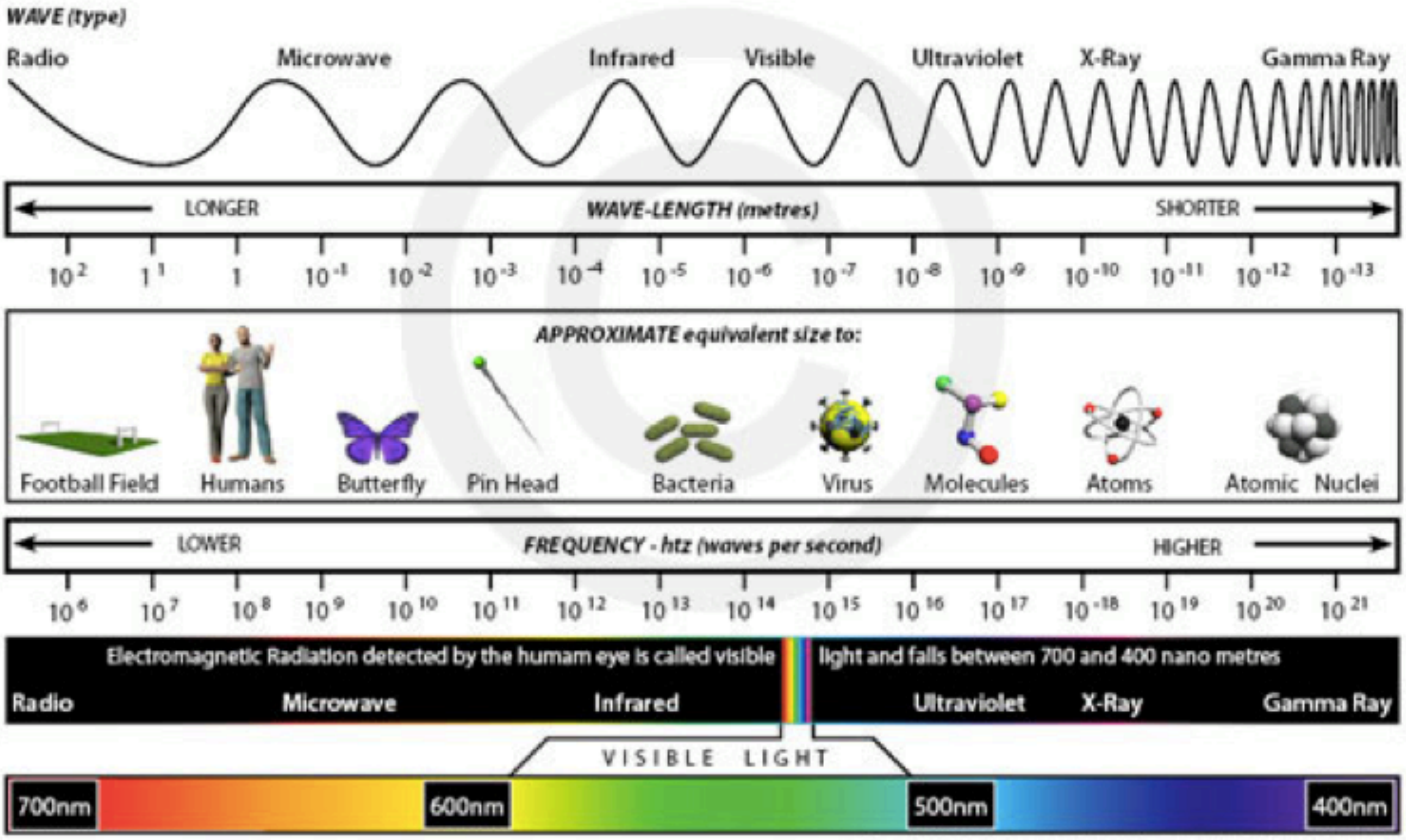
Waves in one-dimension (traveling along *x*)

The Electromagnetic Wave Equations

THE ELECTRO MAGNETIC SPECTRUM

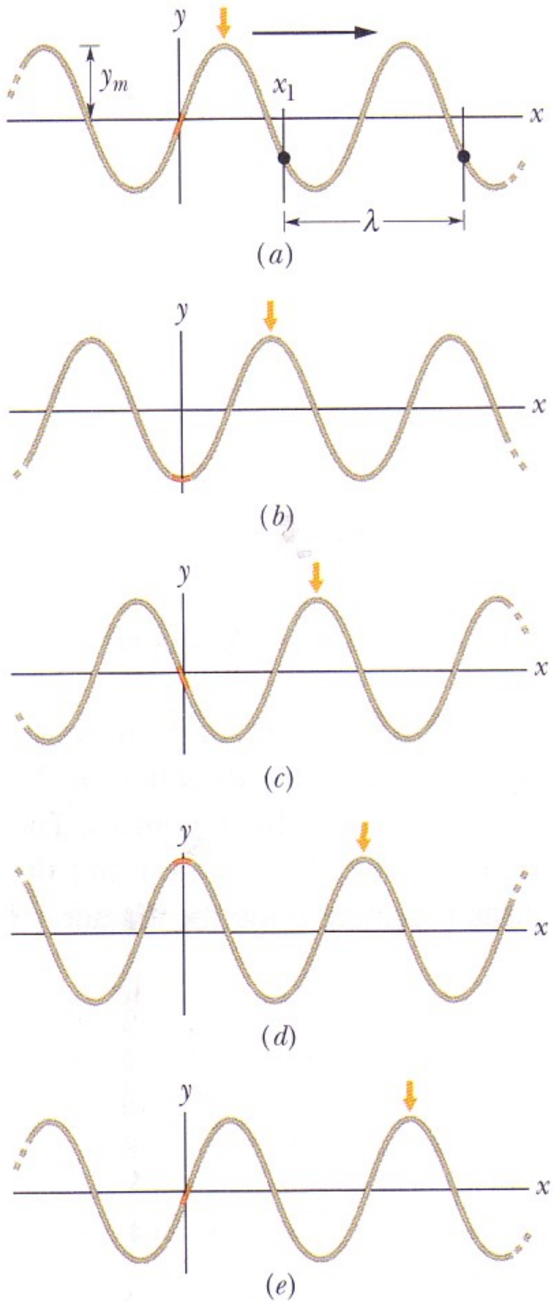
1 metre = 100cm   1 cm = 10mm   1 millimetre = 1000 microns   1 micron = 1000 nanometres (nm) - one nanometer is one billionth of a metre

$10^{-5} = 0.00001$ $10^{5} = 100,000$



Review of waves (PHY2048)

Transverse wave



Displacement

Amplitude

Phase

$$y(x,t) = A\sin(\overbrace{kx \pm \omega t + \phi}^{\text{Phase}})$$

angular wavenumber

angular frequency

Phase shift

$$k = \frac{2\pi}{\lambda}$$

k is the angular wavenumber.

$$\omega = \frac{2\pi}{T}$$

w is the angular frequency.

frequency $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$

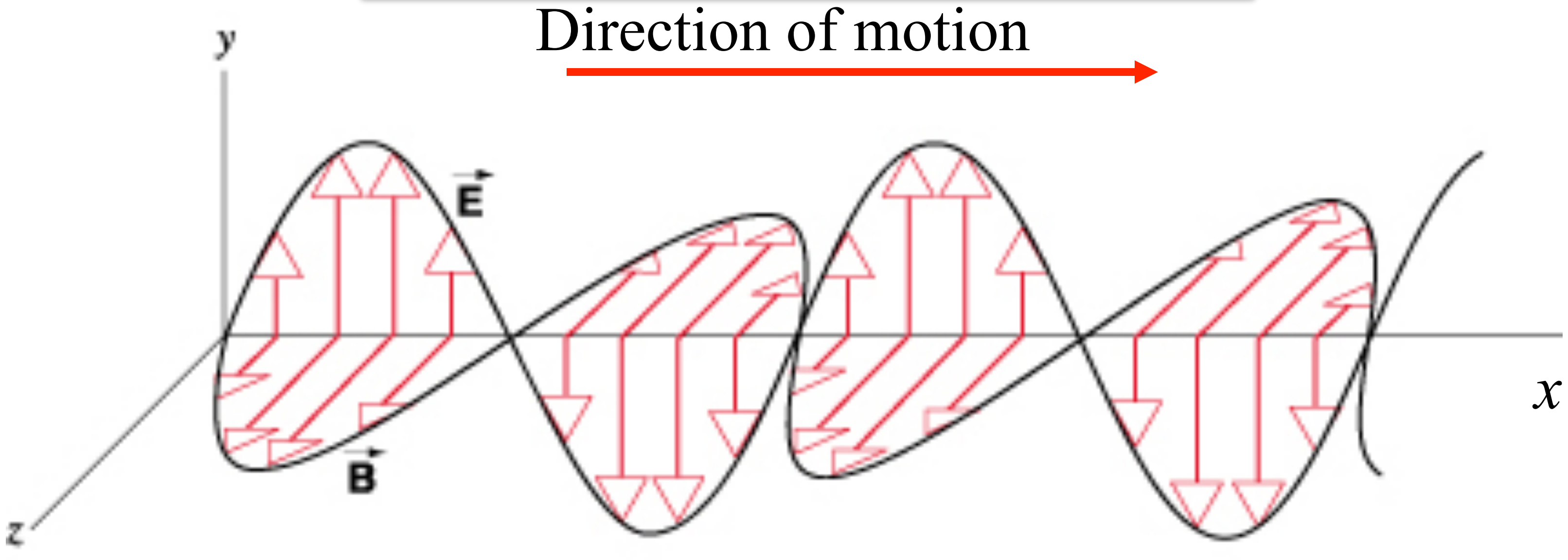
velocity $v = \mp\dfrac{\omega}{k} = \mp\dfrac{\lambda}{T} = \mp f\lambda$

Electromagnetic waves

- The *E* and *B* fields are still related via Ampère's and Faraday's laws.
- For a plane wave traveling in the x direction (see text):

$$\vec{\boldsymbol{E}}\left(x,t\right) = E_{\mathrm{p}} \sin\left(kx - \omega t\right)\hat{\boldsymbol{j}}$$

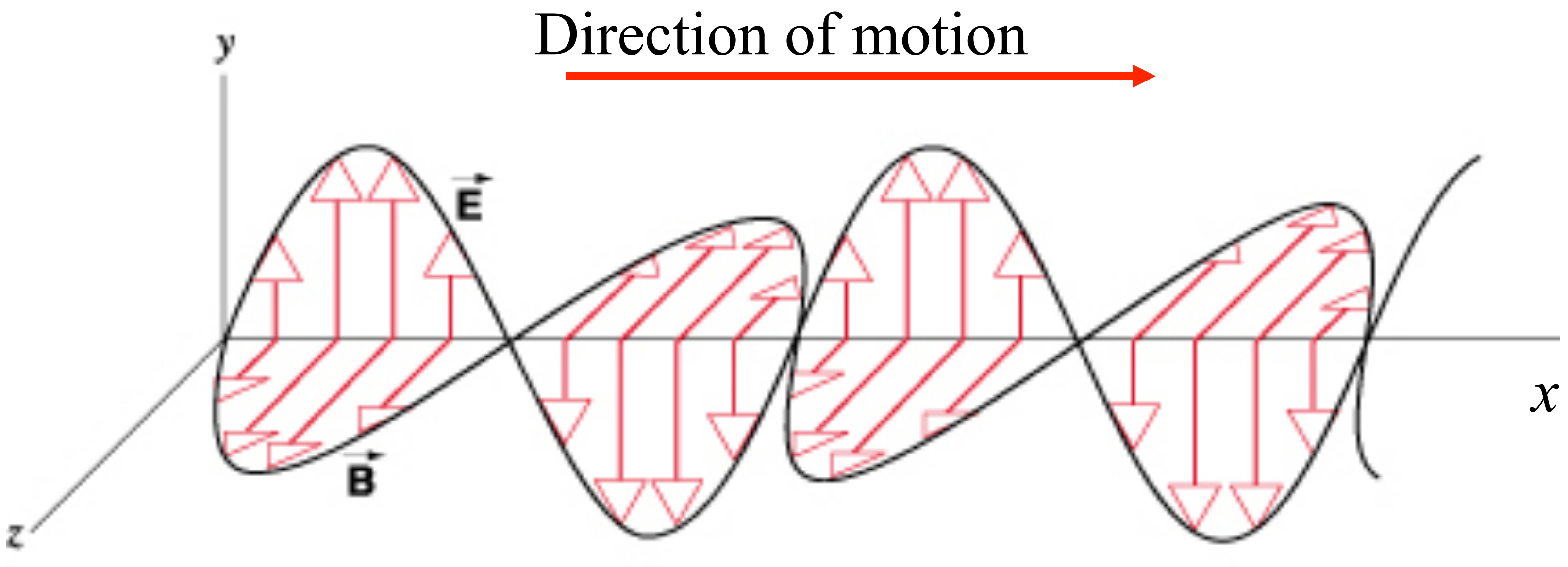
$$\vec{\boldsymbol{B}}\left(x,t\right) = B_{\mathrm{p}} \sin\left(kx - \omega t\right)\hat{\boldsymbol{k}}$$



Electromagnetic waves

- The *E* and *B* fields are still related via Ampère's and Faraday's laws.
- For a plane wave traveling in the x direction (see text):

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}, \qquad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}, \qquad E_x = B_x = 0$$



Electromagnetic waves

- Plugging these wave solutions into the wave equation:

$$\nabla^2 E_y = -k^2 E_y = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2} = -\omega^2 \mu_o \varepsilon_o E_y$$

$$\Rightarrow \boxed{\frac{\omega^2}{k^2} = c^2} = \frac{1}{\mu_o \varepsilon_o}, \qquad \text{or} \qquad \boxed{c = \sqrt{\frac{1}{\mu_o \varepsilon_o}}}$$

- Plugging these wave solutions into Faraday's law:

$$\frac{\partial E_y}{\partial x} = kE_p \cos\left(kx - \omega t\right) = -\frac{\partial B_z}{\partial t} = \omega B_p \cos\left(kx - \omega t\right)$$

$$\boxed{\Rightarrow \frac{E_p}{B_p} = \frac{\omega}{k} = c}$$